

Optimal production management when demand depends on the business cycle

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We assume that consumer demand for an item follows a Brownian motion with drift that is modulated by a continuous-time Markov chain that represents the regime of the economy. The economy may be in either one of two regimes, it remains in one regime for a random amount of time that is exponentially distributed with rate λ_1 , and then moves to the other regime and remains there for an exponentially distributed amount of time with rate λ_2 . Management of the company would like to maintain the inventory level of the item as close as possible to a target inventory level and would also like to produce the items at a rate that is as close as possible to a target production rate. The company is penalized by the deviations from the target levels and the objective is to minimize the total discounted penalty costs over the long term. We consider two models. In the first model the management of the company observes the regime of the economy at all times, whereas in the second model the management does not observe the regime of the economy. We solve both problems and obtain the optimal production policy as well as the minimal total expected discounted cost. Our analytical results show, among various other results, that in both models the optimal production policy depends on factors that are based on short term concerns as well as factors that are based on long term concerns. We analyze how the impact of these factors depend on the values of the parameters in the model. In addition, we compare the total expected discounted costs of the two models with one another and determine the value of knowing the current regime of the economy. We also solve the above problems when the cumulative consumer demand follows a geometric Brownian motion that is modulated by the continuous-time Markov chain that represents the regime of the economy.

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