Stochastic optimization of sailing trajectories in an upwind regatta

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1. In a sailboat regatta, the navigator attempts to plan out the fastest possible course, based on a forecast of wind and weather conditions, as well as to provide real-time updates when these conditions change. It turns out that some wind changes are typically as unpredictable as stock market movements, so it is natural to develop probabilistic models for wind and weather in order to provide tools to help the navigator make decisions.

The velocity of a sailboat that wants to follow a specific bearing is directly influenced by the wind speed and direction. Since a sailboat cannot advance directly into the wind, a boat sailing in an upwind leg must sail at an angle to the wind (of about 30° degrees) and follow a zig-zag trajectory consisting of a sequence of "tackings," that is, of changes in direction such that the bow of the yacht crosses through the eye of the wind. Each tack implies a loss of speed, hence of time, and the decisions concerning when to tack are a crucial part of the navigator's recommendations. Further, since wind direction and speed fluctuate over time, the choice of bearing is difficult to determine intuitively and must be modified during the race.

In this talk, we report on a project to formulate this trajectory optimization problem in the framework of stochastic control theory. The objective was twofold: first, to develop statistical models of wind behavior and use them to perform stochastic optimization under real-world conditions, so as to develop an onboard decision tool that could provide the navigator of the Swiss team Alinghi with real-time recommendations for an America's Cup race, and second, to identify simpler mathematical models of wind behavior that are amenable to a complete mathematical analysis by stochastic optimization methods, with the identification of optimal strategies and rigorous proofs that these strategies are best possible within the model.

2. A statistical analysis of wind behavior was carried out, in collaboration with S. Morgenthaler and S. Sardy at the Ecole Polytechnique Fédérale de Lausanne. Because races occur in the months of May to July, and weather conditions are different during the rest of the year, relevant data can only be collected during these months. Furthermore, in the years that preceded the race, few weather stations were operational and so past data was available only for a limited number of days. Races occur during the afternoon, so a model for wind that would be accurate during a two-hour afternoon race period was needed.

On a race day, the morning's wind could be used to help predict the wind behavior during the afternoon. On the other hand, racing teams were allowed to

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communicate with the outside world only up to five minutes before the race, and after that, could only rely on onboard instruments.

Various continuous-time models have been compared, and turn out to depend on the geographic location: the model we used was different than that suggested in [2]. In the end, for numerical purposes, the evolution of the wind speed and wind direction over time were approximated by discrete-time Markov chains. The transition matrices of these chains were chosen among a small number of possibilities, corresponding to days classified according to low, medium or high volatility of the wind. The data collected on the morning before the race was used to decide the level of volatility for that day. A further effort was made in the last minutes before the break in communication to decide if the wind direction exhibited a trend to the left or right, and the amplitude of that trend.

3. The statistical study produced, for any given day, stochastic processes representing the evolution of wind direction and wind speed over time. In order to carry out numerical computations, the model had to be discretized. Since the position of the boat is essentially a point in the plane, a discretization of the race area was also needed. This discretization had to be compatible with key features of sailboat motion, and fine enough to capture the essential behaviors. In addition, the navigator recommends the bearing that the boat is to follow, so the action space is also continuous, but the model only allowed for a small number of relevant choices. Since the wind direction is measured with a precision of a few degrees, the wind direction was modeled by a discrete-time Markov chain for which each step represented a transition in wind direction and speed over a thirty second time-interval.

4. With the model in hand, offline computations could determine the optimal action for every possible position of the boat on the race field, and every possible wind direction and wind speed. This provided a database that could be used onboard the boat in real time. The onboard computer system could feed in real time wind data, and the model would provide the navigator with a recommendation on the optimal action to be taken. It would also quantify the advantage of using this action relative to other actions, indicate how far ahead one boat is relative to the other, and give advance notice of course changes expected in the near future. These results are reported in [1].

5. In order to carry out a rigorous mathematical analysis, simpler models of wind behavior are useful. This was the objective of [3]. The simplest model for the evolution of wind angle is a two-state continuous time Markov chain. This already leads to a complex free-boundary problem, in which the value function can be written as the solution to a system of hyperbolic partial differential equations with free boundaries, from which many features of the optimal strategy can be obtained. A second natural model is when the wind direction evolves as a Brownian motion on a circle. In this case, the value function solves a system of parabolic partial differential equations with free boundaries. In both cases, the principle of smooth fit can be used to help determine the free boundaries and properties of the value function. This research is reported in [3].

Acknowledgements. This research was supported in part by the Fonds National Suisse pour la Recherche Scientifique.

References

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